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MATHEMATICAL METHODS IN SOME DIFFRACTION PROBLEMS FOR DOMAINS WITH DEFECTS

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Mathematical methods in diffraction problems for elastic time-harmonic waves on defects are considered. It is assumed that the body forces are absent and the defect may be disposed on the plane in the homogeneous isotropic space or on the media interface of two homogeneous isotropic half-spaces. It is obtained systems of singular integral equations equivalent to the problems. Considered mathematical methods may be useful for solving some form researched in [1] diffraction problems for electromagnetic time-harmonic waves on defects. Some approaches to elastodynamic problems in the case of the anisotropic elastic medium are considered too. It is get analogues of the Lopatinskii condition and boundary conditions of an elliptic boundary value problem in the half-space. It is shown that both approaches are equivalent.

To solve these problems *the classes of outgoing from a plane solutions* are introduced. The Fourier transformation in the class of generalized functions of the slow growth at infinity and presentations of solutions of the problems by *potential functions* are used.

SOME ANISOTROPIC ELASTODYNAMIC PROBLEMS.

One considered harmonic oscillations of the anisotropic elastic half-space $\{x_3 > 0\}$. Assume that the body forces are absent. In this case we have the equations

$$\sum_{k,l,j=1}^{3} C_{ijkl} \frac{\partial^2 u_k}{\partial x_l \partial x_j} + \rho v^2 u_i = 0, \quad i = 1,2,3$$

where $u(u_1,u_2,u_3)$ is the complex displacement vector, $\tilde{u}(x,t) = \mathrm{Re}\{u(x)e^{-i\nu t}\}$, $\tilde{u}(\tilde{u}_1,\tilde{u}_2,\tilde{u}_3)$ is the displacement vector, ρ is the density of the body, C_{ijkl} $(i,j,k,l = 1,2,3)$ are the elastic constants, $C_{ijkl} = C_{jikl} = C_{klij}$.

A solution $u(u_1,u_2,u_3)$ of the equation for $x_3 > 0$ we will to call *outgoing from the plane* $\{x_3 = 0\}$ *to the half-space* $\{x_3 > 0\}$ [2], if $u_i(x_1,x_2,x_3)$ $(i = 1,2,3)$ are distributions of the slow growth and

$$supp\ u_i(x_1,x_2,x_3) \subset \{x_3 > 0\}, \quad i = 1,2,3$$

$$sing\,supp\ U_i(\xi_1,\xi_2,\xi_3) \cap \{\xi_3 < 0\} = \varnothing, \quad i = 1,2,3$$

where the Fourier transforms of the unknown functions are denoted by capital letters.

For solving this problem with some boundary conditions on the plane $\{x_3 = 0\}$ the Fourier transformation with respect to all variables in the space of distributions one used. Therefore one get some auxiliary conditions. One can show that the boundary and auxiliary conditions at this approach for solving of the problem are analogous to the Lopatinskii condition and the boundary conditions at the considered in [3] approach for solving of the elliptic boundary value problem for the half-space. In [3] the Fourier

transformation does not take with respect to all variables. Therefore one get a system of ordinary differential equations for the Fourier transforms of unknown functions. But we get the system of linear algebraic equations for the Fourier transforms of $u_i(\cdot,\cdot,\cdot)$ $(i = 1,2,3)$.

If the roots with the positive imaginary part of some equation are known, then the solution of the boundary value problem for the half-space can be written in the obvious form. In this case one can obtain presentations of solutions of the boundary value problems by stress and displacement jumps on the plane $\{x_3 = 0\}$. And it is convenient to research, for example, diffraction problems for the elastic time-harmonic wave on a defect disposed on the plane $\{x_3 = 0\}$.

SOME ISOTROPIC ELASTO- AND ELECTRODYNAMIC PROBLEMS. Let Ω be an infinitely thin defect disposed on the plane $\{x_3 = 0\}$ in an isotropic elastic medium. Assume that the dependence from the time is harmonic for the components of the stress vector and for the components of the displacement vector, the body forces are absent. One searched the complex amplitudes of the functions, the time factor e^{-ikt} one omitted.

It is well known that by made assumptions the elastodynamic equations have the form

$$(\lambda + \mu)\ grad\ div\ u + \mu\Delta u + \rho k^2 u = 0 \quad in \quad R^3 \setminus \overline{\Omega}$$

where $\Delta = \partial^2 / \partial x_1^2 + \partial^2 / \partial x_2^2 + \partial^2 / \partial x_3^2$ is the Laplace operator, λ, μ are the Lame constants, ρ is the density of the body.

In the case of a soldered hard screen, for example, the boundary conditions have the form

$$u_i\big|_\Omega = -u_i^0(x_1, x_2), \quad i = 1,2,3 \quad on \quad \Omega$$

where $u_i^0(\cdot,\cdot)$ $(i = 1,2,3)$ are the known functions.

For solving this problem it is convenient to consider an auxiliary jump problem. One searched solutions of the Lame equations for $\{x_3 > 0\}$ and for $\{x_3 < 0\}$ in the class of solutions outgoing from the plane $\{x_3 = 0\}$. On the plane $\{x_3 = 0\}$ the stress and displacement jumps are given

$$[u_i]\big|_\Lambda = a_{u_i}(x_1, x_2), \quad [\sigma_{i3}]\big|_\Lambda = a_{\sigma_{i3}}(x_1, x_2), \quad i = 1,2,3 \quad on \quad \Lambda$$

where Λ is the plane $\{x_3 = 0\}$, $[f]\big|_\Lambda = f(x_1, x_2, 0+0) - f(x_1, x_2, 0-0)$. Functions in the right-hand sides of the conditions are the given functions on the plane $\{x_3 = 0\}$; we will to call its *the potential functions*.

For solving the jump problem it is convenient to use the longitudinal and lateral potentials $\varphi(\cdot,\cdot,\cdot)$ and $\psi = (\psi_1(\cdot,\cdot,\cdot), \psi_2(\cdot,\cdot,\cdot), \psi_3(\cdot,\cdot,\cdot))$

$$u = \text{grad}\ \varphi + \text{rot}\ \psi, \quad \text{div}\ \psi = 0$$

and the Fourier transformation with respect to all variables in the space of distributions. For the functions $\varphi(\cdot,\cdot,\cdot)$, $\psi_i(\cdot,\cdot,\cdot)$ $(i = 1,2,3)$ we have the Helmholtz equations

$$\Delta\varphi + k_1^2\varphi = 0, \quad \Delta\psi_i + k_2^2\psi_i = 0, i = 1,2,3 \quad in \quad R^3 \setminus \overline{\Omega}$$

where $k_i = k / c_i$ $(i = 1,2)$ $c_1 = \sqrt{(\lambda + 2\mu)/\rho}$, $c_2 = \sqrt{\mu/\rho}$ are the velocities of spreading of longitudinal and lateral waves in the isotropic elastic medium.

In some electrodynamic boundary value problems systems of Helmholtz equations may be obtained too, when boundary conditions for unknown functions do not separate. In these cases solutions of Helmholtz equations we will search independently, if the Fourier transformation in the class of distributions one used.

Boundary conditions for the potentials $\varphi(\cdot,\cdot,\cdot)$ and ψ do not separate in boundary value problems for an isotropic space with the defect Ω on the plane $\{x_3 = 0\}$. But the problems for the Fourier transforms of the functions $\varphi(\cdot,\cdot,\cdot)$ and $\psi_i(\cdot,\cdot,\cdot)$ $(i = 1,2,3)$ in the auxiliary jump problem one can consider independently, if we will take the Fourier transformation in the space of distributions.

For solving the jump problem we take the Fourier transformation with respect to all variables in the Helmholtz equations. One can to obtain presentations of solutions of the diffraction problems for the elastic time-harmonic wave on a defect Ω by the potential functions. One obtained systems of singular integral equations (SSIE) equivalent to the boundary value problems in cases of some defects Ω. For example, one can shown that in the case of a soldered hard screen in presentations of solutions of the boundary value problems the functions $a_{u_i}(\cdot,\cdot)$ $(i = 1,2,3)$ are equal to zero identically and the functions $a_{\sigma_{i3}}(\cdot,\cdot)$ $(i = 1,2,3)$ are non-zero on Ω only. One obtained SSIE for defining the functions $a_{\sigma_{i3}}(\cdot,\cdot)$ $(i = 1,2,3)$ on the screen. Equations of the system have the logarithmic singularity with respect to all variables.

In the considered in Sec.1, 2 dynamic problems longitudinal and lateral potentials are used in the auxiliary jump problem only. It is convenient, because to take the Fourier transformation to the independent Helmholtz equations is more easy than to the system of the connected Lame equations. And the problems for the Fourier transforms of functions $\varphi(\cdot,\cdot,\cdot)$ and $\psi_i(\cdot,\cdot,\cdot)$ $(i = 1,2,3)$ are separated in the jump problem, if one used the Fourier transformation in the space of distributions. Usually in analogous elastodynamic problems the Fourier transformation does not take with respect to all variables. And problems for longitudinal and lateral potentials do not can to consider independently.

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